Dynamical Models for the Milky Way’s Stellar Disk into the Gaia Era

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Milky Way Dynamical Modeling

• How do we learn from the vast ‘phase space data’ about the ‘formation history’ (orbits) and the Galactic potential $\Phi(r)$?

• $\Phi(r)$:
  • What’s the local DM density?
  • Does the dark matter in the MW have a (>kpc) core?
  • Is there a dynamically important dark matter disk?

• Orbits / distribution Function (DF)
  • Age/abundance dependent DF of stars is most fundamental description
  • Need a smooth DF model to quantify deviations from it as sub-structure
  • Does the DF show signs of (impulsive) heating or radial migration?
Milky Way Dynamical Modeling

• How do we learn from the vast ‘phase space data’ about the ‘formation history’ (orbits) and the Galactic potential $\Phi(r)$?

• Given a set of discrete kinematic data, which combinations of DF and $\Phi(r)$ make the data likely?

• Modelling Approaches:
  – Jeans Equation: (+) it’s purely local - you learn nothing about orbits
  – Find joint solutions for orbit distribution (DF) and $\Phi(r)$
    • “analytic” DF: function of integrals of motion or actions
    • Schwarzschild: numerical orbit calculation
    • M2M modelling: iterative N-body scheme
The local dark matter density through the Jeans Equation

a worked example

Kuijken & Gilmore ‘89, Gabari et al ‘11/’12, Bovy & Tremaine ’12, Zhang et al ‘13

• Consider sample of stars \((r,v)\) with good coverage of ‘height above plane’

• Jeans Eq.

\[
\frac{\partial \Phi(R, Z)}{\partial Z} = \frac{1}{\nu} \frac{\partial (\nu \sigma_W^2)}{\partial Z} + \frac{1}{R \nu} \frac{\partial (R \nu \sigma_{UW}^2)}{\partial R} \quad \to \quad \Phi(R_0, z) \to \Sigma_* \text{ and } \rho_{DM}
\]

• Zhang et al 13’: split sample into metallicity bins \(\to\) redundancy & simplicity

• Get good estimates for \(\rho_{DM}\) (but not dramatically better than KG’89!)

Comparison of recent studies

- Tracer density
- Velocity dispersion
- Metal rich
- Metal poor
- \(\chi^2 = 15.90\)
- \(\chi^2_{*} = 26.36\)
Towards a DF & $\Phi(r)$ Galactic Disk Model

Or, how to Dissect the Galactic Stellar Disk?

• The Milky Way’s stellar disk is complex:
  – positions, velocities, ages, abundances are all correlated
    
    $$p(r,v, t_{\text{age}}, [\text{Fe/H}],[\alpha/\text{Fe}],...)$$
  – # of stars with such information e-folding every $\sim 2$ yrs

  • Ground-based surveys: SEGUE, RAVE, APOGEE, PS1, Gaia-ESO, etc. & Gaia

• Let’s separate out the *life-long tags* of stars:

  $$p(\text{Orbits } (r,v) \mid t_{\text{age}}, [\text{Fe/H}],[\alpha/\text{Fe}],...)$$ requires $\Phi(r)$

  Integrals of motion, $E$, $L_z$, ..
  Or .. actions (and angles)
Towards a DF & Φ(r) Galactic Disk Model
Bovy, et al 2012ab, Rix & Bovy ‘13

- Data: SEGUE K or G-dwarf sample (Yanny et al 2009)
  - >10,000 stars with ~10% phot. Distances
  - good [Fe/H],[α/Fe] precision (MS stars of same $T_{\text{eff}}$)

- Let’s first split the sample by same ‘lifelong’ tag...
- Focussing on a “mono-abundance subsets/populations” in [α/Fe] & [Fe/H] space (MAP) ...... what is the spatial structure?
  is the kinematic structure?
The Geometry of Mono-Abundance Sub-Populations

- Fit well by exponential in R and z
- thick = compact = '[\alpha/Fe]-rich'
- thin = radially extended
Abundance-dependent kinematics of the MW Disk
Liu & vdVen 2012, Bovy, Rix et al 2012c

Mono-abundance populations are spatially and kinematically simple
lend themselves to modelling (?)
How to explore “all” DF and $\Phi(r)$ in practice? Parameterize?

- $\text{DF}$ is function of 3 ‘actions’ $\mathcal{J} = (J_R, J_\phi=L_z, J_z)$:

- $L_z$ ($=J_\phi$) sets radial profile, $J_z, J_R$ set $z$ & $R$ ‘temperature’ (McMillan, Binney)
- (only?) MAPs are well-described by the following simple DFs

$$\text{DF}(J_R, L_z, J_z) = \tilde{f}(L_z) \times \exp \left( -\frac{\kappa_R J_R}{\tilde{\sigma}_R^2(R)} \right) \times \exp \left( -\frac{\kappa_z J_z}{\tilde{\sigma}_z^2(R)} \right)$$

$$\tilde{f}(L_z) dL_z = \exp \left( -\frac{R_c}{h_R} \right) R_c dR_c$$
Action-Based Dynamical Modelling

How to explore “all” DF and $\Phi(r)$ in practice? Parameterize?

- DF is function of 3 ‘actions’ $J = (J_R, J_\phi=L_z, J_z)$:
  - $L_z$ sets radial profile, $J_z, J_R$ set $z$ & $R$ ‘temperature’ (McMillan, Binney)
  - (only) MAPs are well-described by these simple DFs

- $\Phi(R,z)$ is bulge + disk + halo

Hernquist

$\rho_d,g(R,Z) \propto \exp \left( -\frac{R}{R_d^{d,g}} - \frac{|Z|}{h_z^{d,g}} \right)$

double exponential for stars & gas

$\rho_h(r) \propto r^{-\alpha}$
Practical Issues in such Dynamical Modelling

- Account for
  - very complex survey volume (DF normalization)
  - discrete tracers with (uneven) individual error bars
- Each MAP will have
  - its own orbit distribution, $DF([\text{Fe/H}],[\alpha/\text{Fe}])$
  - but they all feel the same potential $\Phi(R,z)$
- Computing issues
  - fast DF normalization
  - Fast & accurate $(x,v, \Phi(R,z)) \rightarrow J$
  - complex likelihood surface
What are we learning about the gravitational potential?
(Bovy & Rix 2013)

First dynamical measurement: How much mass is near (~1kpc) the disk plane from R=4-8 kpc

Each MAP*, with a different radial tracer profile, constrains the disk mass density (best) at a different R!

$\rho_{DM}(r) \propto r^{-\alpha_h}$

* MAP: mono-abundance stellar population
What are the important limitations?

At the latest after Gaia L+28: vast samples of tracers with sufficiently precise \((r,\mathbf{v})\) → all limitations that scale with \((N_{\text{sample}})^{-1/2}\) are irrelevant

- Can (sub-populations of) stars be well described by simple distribution functions?
  Do we have modelling techniques that scale to vast \(N_{\text{sample}}\)?

- Can we control/understand the selection function?

  **spatial selection** \(v_{\text{tracer}}(r)\)
  – “positions” are as important as “motions” in dynamics!!
  – dust extinction modifies \(f \sim 1/D^2\)

  **abundance selection** \(p_{\text{select}}([\text{Fe/H}], [\alpha/\text{Fe}], \text{etc}..)\)
  – “mix” of abundances must not change (or must be modeled) with orbital phase

- The Milky Way is neither (exactly) axisymmetric nor in (exact) equilibrium
Importance of knowing the spatial distribution of the tracers

• Consider simple example: spherical

\[ M(r) = -\frac{r \sigma_{rr}^2}{G} \left[ \frac{d \ln \nu}{d \ln r} + \frac{d \ln \sigma_{rr}^2}{d \ln r} + 2\beta(r) \right] \]

\[ \beta \equiv 1 - \frac{\sigma_{l1}^2}{\sigma_{rr}^2} \]

Overall scaling

tracer gradient

\[ \sim -3 \]

kinematic gradient

\[ \sim -0.3 \]

orbit anisotropy

\[ \sim 0.3 \]

\[ \rightarrow \text{(systematic) uncertainty in tracer gradient may be dominant} \]

• Current application: what is the Milky Way’s halo mass?

Battaglia et al 05
Xue et al 08,15
Deason et al 11ab, 14
Kafle et al 12,14

Xue et al 2015
Know thy spatial tracer distributions:
High-resolution 3D dust maps of the Galaxy

- PanSTARRS1: a high precision (0.5%) multi-color survey across ¾ of the sky
- Joint distance $D$ and $A_v$ estimate for a billion main-sequence stars

- Combine adjacent stars to get $A_v(D) \rightarrow 3D$ extinction map

Schlafly et al 2014
Green et al 2014

star-by-star-derived 3D extinction map to successively larger distances (PS1 data)
The need for dust extinction correction

Schlafly et al 2014
Green et al 2014

Color: distance [kpc] to $A_R=1$

Quality of the ‘vertical tracer density profile’ will be determined by the quality of the 3D (de-)reddening model
also the kinematics of disk stars are very abundance-dependent

→ must know the spatial dependence of metallicity selection

**NB:** $p([\text{Fe/H}],[a/\text{Fe}],..)$ need not be $\delta([\text{Fe/H}],[a/\text{Fe}],..)$ i.e. mono-abundance

.. but that’s convenient
What will Gaia bring immediately? What needs attention?

• What Gaia will bring immediately?
  – proper motions beyond $\sim$1-2 kpc
    • distant $z$-velocities are almost all “proper motions”
  – calibration of distances and 3D map
    • distance *calibration* more important (>1-2 kpc) than parallaxes?

• What needs (more) attention?
  – incorporation of complex spatial/”abundance” selection function in any form of dynamical modeling!
  – dynamical modeling with orbits that scales to large $N_\ast$