ASTROSTATISTICS FOR LUMINOSITY CALIBRATION IN THE GAIA ERA

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INTRODUCTION

• Gaia will provide measurements of observables (parallax, apparent magnitude, etc.).

• It will be essential to use derived quantities (distance, absolute magnitude, etc.).

• There are complications due to non-linear transformations, sample selection effects and statistical biases.

• A reliable method for obtaining derived quantities is through statistical inference.
TRANSFORMATIONS

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Example Monte Carlo simulation for a single star
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Example Monte Carlo simulation for a single star

\[ M = m + 5 \log(\varpi) + 5 \]
SELECTION EFFECTS

Selection effects and biases must be accounted for.
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MODEL THE DATA

\[ P(A|B) = \frac{P(B|A)P(A)}{P(B)} \]

A is the set of parameters describing a system
B is the data drawn from the system
MODEL THE DATA

\[ \mathcal{P}(x|\theta) = C^{-1} \varphi_M(m) \varphi_r(\varpi, l, b) S(m) \]

A list of stars

\[ x = (x_1, x_2, \ldots, x_N) \]

with observables

\[ x_i = (m, \varpi, l, b) \]

and the model parameters

\[ \theta = (M_{\text{mean}}, \sigma_M) \]
MODEL THE DATA

PDF of magnitudes

\[ \mathcal{P}(x|\theta) = C^{-1}\varphi_M(m)\varphi_r(\varpi, l, b)S(m) \]

PDF of distances

including the spatial distribution of the stars accounts for the Lutz-Kelker Bias

The selection of stars due to a limiting magnitude can also be taken into account (Malmquist bias)
MODEL THE DATA

PDF of ‘real’ magnitudes

$$
\mathcal{P}(x_i | \theta) = C^{-1} \mathcal{S}(x) \int_{\forall x_0} \varphi_{m_0}(m_0) \varphi_{\nu}(\nu_0, l_0, b_0) \mathcal{E}(x | x_0) \, dx_0
$$

Marginalise the unknown ‘real’ values

Link the observations to the true values via the error function

PDF of ‘real’ distances
APPLICATION: DISTANCE TO THE LMC

\[
\ln \mathcal{L}(\theta | x) = \sum_{i=1}^{N} \ln \left( \frac{1}{2\pi \sigma_d \epsilon_i} \right) - 0.5 \left( \frac{d_{r,i} - d_{\text{mean}}}{\sigma_d} \right)^2 + \left( \frac{\omega_i - 1/d_{r,i}}{\epsilon_i} \right)^2
\]

Distribution of 'real' distances

Normalisation

Link the observed parallax with the true distance, without non-linear transformations on error effected variables
APPLICATION: DISTANCE TO THE LMC

Mean distance determination for the LMC is possible to within 0.5%.

Expected relative error distribution for all Gaia LMC stars.
APPLICATION: OPEN CLUSTERS

Spherical spatial distribution

From Palmer et al. 2014
APPLICATION: OPEN CLUSTERS

Magnitude distribution following isochrone

From Palmer et. al. 2014
APPLICATION: VARIABLE STARS

Magnitude distribution following a Period-Luminosity relation

Synthetic data for LMC Cepheid population (based on VMC/OGLE data)
CONCLUSIONS

• Remember that inverting the parallax can lead to biases

\[ \langle d \rangle \neq \langle \frac{1}{\sigma} \rangle \neq \frac{1}{\langle \sigma \rangle} \]

• Posterior corrections for Malmquist and Lutz-Kelker biases are highly imprecise

• Instead - model the data and infer derived quantities